**Yashika LNU**

**G21358006**

**Question 1- Recommendation System**

a) The non-normalized user similarity matrix T is defined as:

T = R . RT

Where:

* R is the user-item binary matrix,
* RT is the transpose of R,
* T[i][j] is the dot product of user i and user j's preference vectors

Interpretation:

* Tᵢᵢ (diagonal elements): Represents the total number of items liked by user i.
* Tᵢⱼ (off-diagonal elements, where i ≠ j): Represents the number of items commonly liked by both user i and user j.

For the given example,

Cosine Similarity:

To normalize the similarity, we use cosine similarity:

This gives a similarity score between 0 and 1 that accounts for how many items users like in common, adjusted for how many total items they each liked.

## b)

## **Item Similarity Matrix (Sᴵ)**

We are given:  
  
Where:  
- Rᵀ R computes the non-normalized item-item similarity (dot product between item vectors).  
- Q is a diagonal matrix where Q[i][i] is the number of users who liked item i.  
- Q^(-1/2) is a diagonal matrix with 1/sqrt(Q[i][i]) for non-zero entries, and 0 otherwise.  
  
This results in cosine similarity, since:

and applying Q^(-1/2) on both sides normalizes the dot product accordingly.

## **User Similarity Matrix (Sᵁ)**

We are given:  
  
Where:  
- R \* Rᵀ computes the non-normalized user-user similarity (dot product between user vectors).  
- P is a diagonal matrix where P[i][i] is the number of items liked by user i.  
- P^(-1/2) is a diagonal matrix with for non-zero entries, and 0 otherwise.  
  
This computes cosine similarity between users, as:

And the formula:

applies that normalization directly, yielding cosine similarities.

## **Verification with Example**

Given matrix R:

Then:

Compute:  
  
We already computed:

Then Item Similarity Matrix (Sᴵ):

This matches the cosine similarity matrix for items.  
  
Similarly, we computed:

correctly produce cosine similarity matrices for items and users, respectively.

c)

We are tasked with deriving the recommendation matrix Γ ( m×n), where Γi,j represents the predicted preference score of user i for item j, for both user-user and item-item collaborative filtering methods. Let’s define and justify the expressions based on the given formulas.

#### **User-User Collaborative Filtering**

The predicted rating for user u and item s is:

This computes as a weighted sum of item s’s ratings across all users x, with weights given by the cosine similarity between users x and u.

From part (b), the user similarity matrix is:

This can be interpreted as:

1. Use cosine similarities between users as

1. Multiply with R to get predicted ratings:

So:

#### **Item-Item Collaborative Filtering**

We compute

This can be interpreted as:

1. Use cosine similarities between items:
2. Multiply R with ​ to get predicted ratings: